

二阶半线性泛函微分方程解的振动性

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摘要: 通过引入辅助函数和利用 Young 不等式技巧, 研究二阶半线性泛函微分方程解的振动性, 所得的结果是新的, 且改进了 Agawal R P 等人的一个结果.

关键词: 振动; 二阶半线性泛函微分方程; 解

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本文研究二阶半线性泛函微分方程

$$[|y'(t)|^{p-1}y'(t)]' + q(t)|y(h(t))|^{p-1}y(h(t)) = 0, \quad (1)$$

在区间 $[t_0, \infty)$ 上解的振动性质. 这里我们恒设 $p > 0$ 是一个常数, $q \in C'([t_0, \infty), [0, \infty))$, $h \in C([t_0, \infty), R)$, $h(t) \leq t$, 且 $\lim_{t \rightarrow \infty} h(t) = \infty$.

称函数 $y \in C'([T_y, \infty), R)$ (对 $T_y \geq t_0$) 为方程(1)的解, 如果且有属性 $|y'(t)|^{p-1}y'(t) \in C'([T_y, \infty), R)$ 并且满足方程(1). 本文仅考虑方程(1)满足 $\sup\{|y(t)| : t \geq T\} > 0$ (对所有 $T \geq T_y$). 以后我们总是假设方程(1)在 $t \geq t_0$ 上存在这样的解. 方程(1)的一个解称为振动, 如果它有充分大的零点, 否则称之为非振动.

方程(1)对应的常微分方程 ($h(t) \equiv t$), 其解的振动性质已被众多学者所研究, 例见文献 [1~5] 及其所列文献. 最近, 文献[6] 获得如下定理 (参见文献[6] 定理 2.8).

定理 A 若

$$\int_{t_0}^{\infty} q(s) \left[\frac{h(s)}{s} \right]^p ds = \infty, \quad (2)$$

则方程(1)的每个解是振动的.

本文的目的是通过引入辅助函数和利用 Young 不等式技巧, 建立方程(1)的每个解是振动的新的充分条件, 并改进定理 A. 最后给出几个例子, 以说明本文结果的优越性.

定理 1 若存在函数 $\varrho, \varphi \in C'([t_0, \infty), R^+ = (0, \infty))$ 和常数 $k \in (0, 1)$, 使得

$$\limsup \int \left\{ \varphi(s) \varrho(s) q(s) \left[k \frac{h(s)}{s} \right]^p - \frac{1}{(p+1)^{p+1}} \frac{\varrho(s)}{[\varphi(s)]^p} [|\varphi'(s)| + \varphi(s) \frac{|\varrho'(s)|}{\varrho(s)}] J^{p+1} \right\} ds = \infty, \quad (3)$$

则方程(1)的每个解是振动的.

证 设 $y(t)$ 是方程(1)的一个非振动解. 不失一般性, 不妨设 $y(t) > 0$ 且 $y(h(t)) > 0$, 对 $t \geq t_1 \geq t_0$. 类似于文献[6] 定理 2.3 的证明有 $y'(t) > 0$, $y''(t) < 0$, 对 $t \geq t_2 \geq t_1$, 则依文献[6] 引

理 2.1(a) 知, 对任意给定常数 $k \in (0, 1)$, 存在 $t_3 \geq t_2$, 使得

$$y(h(t)) \geq k \frac{h(t)}{t} y(t), \text{ 对 } t \geq t_3. \quad (4)$$

令

$$W(t) = \rho(t) \frac{|y'(t)|^{p-1} y'(t)}{|y(t)|^{p-1} y(t)} \quad \text{对 } t \geq t_3, \quad (5)$$

很明显 $W(t) > 0$. 由(1)和(5)并注意到式(4), 得

$$\begin{aligned} W'(t) &= \frac{\rho'(t)}{\rho(t)} W(t) - \rho(t) q(t) \frac{|y(h(t))|^{p-1} (h(t))}{|y(t)|^{p-1} y(t)} - p \frac{[W(t)]^{\frac{p+1}{p}}}{[\rho(t)]^{\frac{1}{p}}} \\ &\leq \frac{\rho'(t)}{\rho(t)} W(t) - \rho(t) q(t) [k \frac{h(t)}{t}]^p - p \frac{[W(t)]^{\frac{p+1}{p}}}{[\rho(t)]^{\frac{1}{p}}}, \quad \text{对 } t \geq t_3. \end{aligned} \quad (6)$$

对上式两边同乘以 $\varphi(t)$, 并对其从 t_3 到 t 积分之, 有

$$\begin{aligned} \int_{t_3}^t \varphi(s) \rho(s) [k \frac{h(s)}{s}]^p ds &\leq \int_{t_3}^t \varphi(s) W' ds + \\ \int_{t_3}^t \varphi(s) \frac{\rho'(s)}{\rho(s)} W(s) ds - p \int_{t_3}^t \varphi(s) \frac{[W(s)]^{\frac{p+1}{p}}}{[\rho(s)]^{\frac{1}{p}}} ds &\leq \\ - \varphi(t) W(t) + \varphi(t_3) W(t_3) + \int_{t_3}^t \phi(s) W(s) ds + \\ \int_{t_3}^t \varphi(s) \frac{|\rho'(s)|}{\rho(s)} W(s) ds - p \int_{t_3}^t \varphi(s) \frac{[W(s)]^{\frac{p+1}{p}}}{[\rho(s)]^{\frac{1}{p}}} ds. \end{aligned} \quad (7)$$

依 Young 不等式

$$\begin{aligned} [|\phi(s)| + \varphi(s) \frac{|\rho'(s)|}{\rho(s)}] W(s) &= \left\{ \frac{(p+1) \varphi(s)}{[\rho(s)]^{\frac{1}{p}}} \right\}^{\frac{p}{p+1}} W(s) \left\{ \frac{(p+1) \varphi(s)}{[p(s)]^{\frac{1}{p}}} \right\}^{-\frac{p}{p+1}} [|\phi(s)| + \\ \varphi(s) \frac{|\rho'(s)|}{\rho(s)}] &\leq p \frac{\varphi(s)}{[\varphi(s)]^{\frac{1}{p}}} [W(s)]^{\frac{p+1}{p}} + \frac{1}{(p+1)^{p+1}} \frac{\rho(s)}{[\varphi(s)]^p} [|\phi(s)| + \varphi(s) \frac{|\rho'(s)|}{\rho(s)}]^{p+1}. \end{aligned} \quad (8)$$

把(8)代入(7), 得

$$\int_{t_3}^t \left\{ \varphi(s) \rho(s) q(s) [k \frac{h(s)}{s}]^p - \frac{1}{(p+1)^{p+1}} \frac{\rho(s)}{[\varphi(s)]^p} [|\phi(s)| + \varphi(s) \frac{|\rho'(s)|}{\rho(s)}]^{p+1} \right\} ds \leq \varphi(t_3) W(t_3).$$

对上式两边取上极限, 得出与(3)相矛盾. 证毕.

定理 2 设 $p > 1$. 若存在函数 $\rho, \varphi \in C'([t_0, \infty), R^+)$, 使得

$$\rho(s) [|\phi(s)| + \varphi(s) |\rho'(s)|] \quad \text{不增, 对充分大的 } t, \quad (9)$$

且

$$\int_{t_3}^{\infty} \varphi(s) \rho(s) q(s) [k \frac{h(s)}{s}]^p ds = \infty, \quad (10)$$

则方程(1)的每个解是振动的.

证. 类似于定理 1 的证明知(6)成立. 于是

$$\rho(t)q(t)\left[\frac{h(t)}{t}\right]^p \leq -W'(t) + \frac{\rho'(t)}{\rho(t)}W(t), \quad \text{对 } t \geq t_3.$$

对上式两边同乘以 $\varphi(t)$, 并对其两边从 t_3 到 t 积分之, 有

$$\begin{aligned} \int_{t_3}^t \varphi(s)\rho(s)q(s)\left[\frac{h(s)}{s}\right]^p ds &\leq \int_{t_3}^t \varphi(s)W'(s)ds + \int_{t_3}^t \varphi(s)\frac{\rho'(s)}{\rho(s)}W(s)ds = \\ &= -\varphi(t)W(t) + \varphi(t_3)W(t_3) + \int_{t_3}^t \varphi'(s)W(s)ds + \int_{t_3}^t \varphi(s)\frac{\rho'(s)}{\rho(s)}W(s)ds \leq \\ &= \varphi(t_3)W(t_3) + \int_{t_3}^t \frac{|\rho(s)| + |\varphi'(s)| + |\varphi(s)| + |\rho'(s)|}{\rho(s)}W(s)ds = \\ &= \varphi(t_3)W(t_3) + \int_{t_3}^t \left[|\rho(s)| + |\varphi'(s)| + |\varphi(s)| + |\rho'(s)|\right] \frac{[\gamma'(s)]^p}{[\gamma(s)]^p} ds. \end{aligned} \quad (11)$$

注意到 $\gamma'(t)$ 的单减性, 则存在常数 $C > 0$, 使得 $\gamma'(t) \leq C$, 对 $t \geq t_3$. 于是(11)变形为

$$\begin{aligned} \int_{t_3}^t \varphi(s)\rho(s)q(s)\left[\frac{h(s)}{s}\right]^p ds &\leq \varphi(t_3)W(t_3) + \\ &+ C^{p-1} \int_{t_3}^t \left[|\rho(s)| + |\varphi'(s)| + |\varphi(s)| + |\rho'(s)|\right] \frac{\gamma'(s)}{[\gamma(s)]^p} ds. \end{aligned} \quad (12)$$

依 Bonnet 定理, 并注意到(9), 对任意固定的 $t \geq t_3$, 存在 $\zeta \in [t_3, t]$, 使得

$$\begin{aligned} \int_{t_3}^t \left[|\rho(s)| + |\varphi'(s)| + |\varphi(s)| + |\rho'(s)|\right] \frac{\gamma'(s)}{[\gamma(s)]^p} ds &= \\ \left[|\rho(t_3)| + |\varphi'(t_3)| + |\varphi(t_3)| + |\rho'(t_3)|\right] \int_{t_3}^{\zeta} \frac{\gamma'(s)}{[\gamma(s)]^p} ds &= \\ \frac{1}{p-1} \left[|\rho(t_3)| + |\varphi'(t_3)| + |\varphi(t_3)| + |\rho'(t_3)|\right] \left[\gamma^{1-p}(t_3) - \gamma^{1-p}(\zeta)\right] &\leq \\ \frac{1}{p-1} \left[|\rho(t_3)| + |\varphi'(t_3)| + |\varphi(t_3)| + |\rho'(t_3)|\right] \gamma^{1-p}(t_3) &= M < \infty. \end{aligned} \quad (13)$$

由(12)与(13)得

$$\int_{t_3}^t \varphi(s)\rho(s)q(s)\left[\frac{h(s)}{s}\right]^p ds \leq k^{-p} [\varphi(t_3)W(t_3) + C^{p-1}M] < \infty.$$

这与(10)相矛盾. 证毕.

定理 3 若存在函数 $\rho, \varphi \in C'([t_0, \infty), R^+)$, 使得

$$\int_{t_0}^{\infty} \left[|\rho(s)| + |\varphi'(s)| + |\varphi(s)| + |\rho'(s)|\right] ds < \infty, \quad (14)$$

且

$$\int_{t_0}^{\infty} \varphi(s)\rho(s)q(s)\left[\frac{h(s)}{s}\right]^p ds = \infty, \quad (15)$$

则方程(1)的每个解是振动的.

证 类似于定理 2 的证明知(11)成立. 注意到 $\gamma'(t)$ 的单减性和 $\gamma(t)$ 的单增性, 则存在常数 $C > 0$, 使得 $\left[\frac{\gamma'(t)}{\gamma(t)}\right]^p \leq C$, 对 $t \geq t_3$. 于是, 由式(11)与(14), 得

$$\int_{t_3}^t \varphi(s) \rho(s) q(s) \left[\frac{h(s)}{s} \right]^p ds < k^{-p} [\varphi(t_3) W(t_3) + C \int_{t_3}^t [\rho(s) |\varphi'(s)| + \varphi(s) |\rho'(s)|] ds] < \infty.$$

这与(15)相矛盾. 证毕.

注1 在定理1-3中, 若取 $\rho(t) = \varphi(t) = 1$, 则定理A成立.

例1 考虑方程

$$[|y'(t)| + y'(t)]' + t^{-2} |y(\frac{t}{2})| y(\frac{t}{2}) = 0, \quad t > 0, \quad (16)$$

这里 $p = 2$, $q(t) = t^{-2}$, $h(t) = \frac{t}{2}$. 取 $\rho(t) = 1$, $\varphi(t) = 1$, 则对任意给定常数 $k \in (0, 1)$, 有

$$\limsup_t \int_t^t \{ \varphi(s) \rho(s) q(s) \left[\frac{h(s)}{s} \right]^p - \frac{1}{(p+1)^{p+1}} \frac{\rho(s)}{[\varphi(s)]^p} [|\varphi'(s)| + \varphi(s) \left| \frac{\rho'(s)}{\rho(s)} \right|]^{p+1} \} ds = \limsup_t \int_t^t \left[\left(\frac{k}{2} \right)^2 \frac{1}{s} - \frac{1}{27} \frac{1}{s} \right] ds = \infty,$$

则依定理1知方程(16)的每个解是振动的.

例2 考虑方程

$$[|y'(t)|^{p-1} y'(t)]' + t^{-p} |y(\frac{t}{2})|^{p-1} y(\frac{t}{2}) = 0, \quad t > 0, \quad (17)$$

这里 $1 < p \leq 2$, $q(t) = t^{-p}$, $h(t) = \frac{t}{2}$. 取 $\rho(t) = t^p$, $\varphi(t) = t^{-1}$, 则

$$\rho(s) |\varphi'(s)| + \varphi(s) |\rho'(s)| = (p+1) t^{p-2} \text{ 不增}$$

且

$$\int_t^\infty \varphi(s) \rho(s) q(s) \left[\frac{h(s)}{s} \right]^p ds = \int_t^\infty \left(\frac{1}{2} \right)^p \frac{1}{s} ds = \infty,$$

则依定理2知方程(17)的每个解是振动的.

注2 不难看出定理A对例1与例2失效.

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Oscillation for Second Order Half-linear Functional Differential Equations

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Abstract: By means of auxilliary functions and Young inequality technique, new oscillation criteria are established for second order half-linear functional differential equations. Its results extend and improve the oscillation theorem of Agarwal R. P. et al.

Key words: oscillation; second order half linear functional differential equations; Young inequality

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Software Analysis of Sound Image Trait for 5.1 Channel Surround Sound System

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Abstract: By using a newly-made software for stereophonic and surround sound image analysis, this paper analyzes sound image localization trait for 5.1 channel surround sound system. The result is consistent with that of computation based on surround sound image localization formular. The software also evaluates the sound image clearness and naturalness of the system.

Key words: 5.1 channel; sound image localization; clearness; naturalness